Co-developing beliefs and social influence networks
– *towards understanding Brexit*

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Abstract. A relatively simple model is presented where the beliefs of agents and their social network co-develop. Agents can either hold or not each of a fixed menu of candidate beliefs. Depending on their type, agents have different coherency functions between beliefs, so that they are more likely to adopt a belief from a neighbour or drop a belief where this increases the total coherency of their belief set. With given probabilities links are randomly dropped or added but, if possible, links are made to a "friend of a friend". The outcomes when both belief and link change processes occur are qualitatively different from either alone, showing the necessity of representing both cognitive and social processes together. Some example results are shown which moves a little towards modelling the processes behind divisive collective decisions, such as the Brexit vote.

1. MOTIVATION

Like many in the UK, I was shocked at the outcome of the Brexit vote. I am still in a kind of mourning, having lost the country I thought I was living in. Thus, it was natural to me to want to understand the processes that led to this. This paper describes the first tentative steps in that direction.

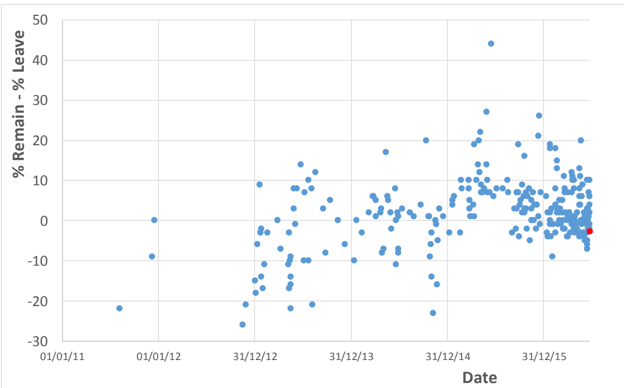

Figure 1. %Remain – %Leave in polls over time, leading up to Brexit vote (red dot is final result)

Let us look at some of the data for clues about the kind of processes that might be involved. Figure 1 shows the raw %Remain – %Leave difference in polls leading up to the final vote. However, this graph is somewhat difficult to read. Figure 2 shows the same data, but (a) adjusted for known biases between telephone and online polls and (b) exponentially smoothed using a factor of 25%. This shows definite medium-term trends, but also a lot of short term 'noise'.

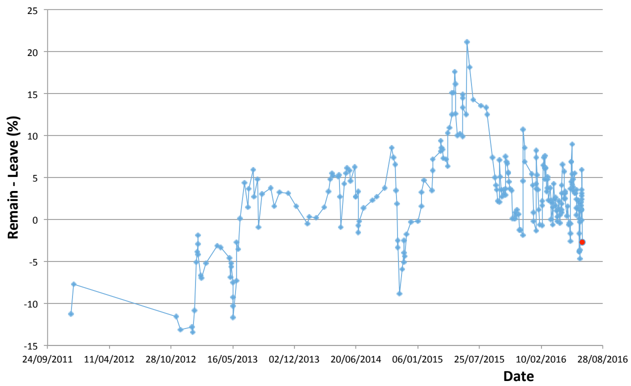

Figure 2. Adjusted, smoothed opinion poll differences between Remain and Leave (%) during run-up to Brexit vote, adjusted for known biases in online and telephone polls.

One of the factors that graphs, such as the above, leave out is that a significant proportion of those polled are undecided. These people are important, because votes are often won by persuading the supporters of the other side to stay and home and not vote, as much as persuading the undecided to vote for yourself.

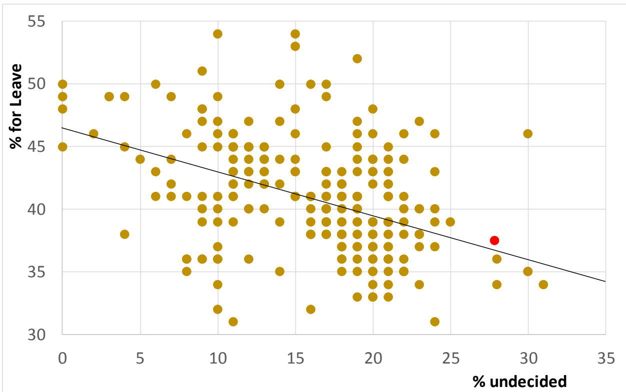

Figure 3. % for leaving vs. % undecided for opinion polls during run-up to Brexit vote, with linear regression line. (red dot is final vote, mapping non-voters to undecided)

Figure 3 shows each poll plotted in terms of the proportion for leaving against the proportion undecided. These differ greatly in terms of the proportion undecided because each poll is designed very differently in terms of how easy it is to chose undecided as an option. Some polls strongly encourage users to make a decision (e.g. by not offering it as an option). Figure 4 is the equivalent graph but for remaining against undecided. One can see that the final result is consistent with the polls if one takes the undecided into account. If there were no undecided, this data suggests that the result might have been different.

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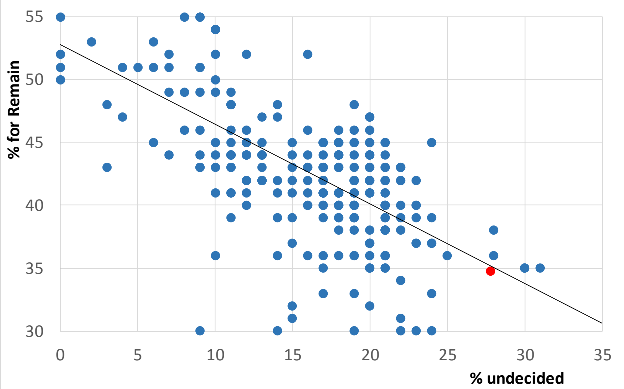

Figure 4. % for remaining vs. % undecided for opinion polls during run-up to Brexit vote, with linear regression line. (red dot is final vote, mapping non-voters to undecided)

Turning to the structure of social influence, there is no doubt that the social network along which influence can occur is clustered into those with similar beliefs. However, direct evidence for the structure of this is hard to come by and we have to be satisfied with indirect indications. Krasodomski-Jones of Demos [6] selected 2500 at random from a larger population of political twitter accounts, divided equally into Labour, SNP, Tory and UKIP supporters (as well as a control group that I do not include here). It then analysed over a million tweets from these users from May-August 2016. One of these analyses was to see who re-tweeted tweets from whom. Table 1 shows a summary of this analysed by the party they support. They then visualised the re-tweet network in a similar manner to [1].

Table 1. Percentage of users re-tweeted by user group, from [6]. (>40% in orange, ≤10% in blue, rest in green)

		Users Re-tweeted			
		Labour	SNP	Tory	UKIP
Users Re-tweeting	Labour	65%	12%	14%	6%
	SNP	18%	78%	8%	3%
	Tory	12%	5%	46%	18%
	UKIP	6%	4%	32%	73%

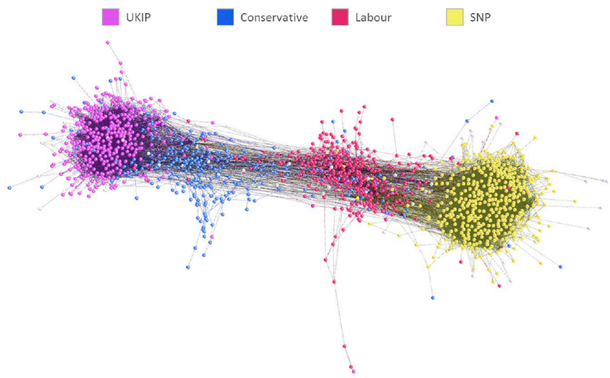

Figure 5. The re-tweet network between 4 groups of 500 supporters of four UK parties, from [6].

The picture that emerges is that these actors are very much sorted by (a) their own party and (b) on a roughly linear political spectrum. That is most re-tweets were within their own party. There was some re-tweeting between: UKIP & Tory, Tory & Labour, Labour & SNP, but very low levels of re-tweeting between SNP and either UKIP or Tory or between UKIP and either SNP or Labour.

Some lessons I take from these include, that:

- opinion formation is noisy and not smooth,
- undecided actors matter almost as much as those of decided or fixed opinion, and
- people communicate with (and hence tend to influence) those similar to themselves, and in particular do not link to those with dissimilar beliefs.

2. THEORETICAL BACKGROUND

The "Social Intelligence Hypothesis" (SIH) [7] states that the *crucial* evolutionary advantages that human intelligence gives are due to the social abilities it allows. This explains specific abilities such as: imitation, language, social norms, lying, alliances, gossip, politics, group identification etc. Under this view, social intelligence is ***not*** a result of general intelligence being applied to social matters, but at the core of human intelligence. One might even go as far as saying that so-called "general intelligence" is a side-effect of social intelligence (e.g. linguistic ability and the ability to learn and beliefs from others).

Consistent with SIH is the following evolutionary story. Social intelligence allows humans to develop their own (sub) cultures of knowledge, technologies, norms etc. (Boyd and Richerson 1985) within their groups. These allow the groups *with their culture* to inhabit a variety of ecological niches (e.g. the Kalahari, Polynesia) (Reader 1980). Thus humans, as a species, are able to survive catastrophes that effect different niches in different ways (specialisation) since it is unlikely that all inhabited niches will be wiped out.

This means that different "cultures" of knowledge, skill, habits, norms, narratives etc. are significant (including how to socially organise, behave, coordinate etc.), and that these will relate to each other as a complete "package" to a significant extent. Under this view human cognition is (at least partly) evolved to make this group survival work, including the capacity of maintaining a complex set of beliefs that are coherent with others in the group, whilst retaining flexibility. The model described herein exhibits such a combination of local coherency and global variety corresponding to the social structure.

Granovetter [5] Contrasted both under- and over-socialised models of behaviour of human behaviour – criticising sociologists in their picture of humans as culturally determined, and economists for their picture of agents characterised by self-interested behaviour. That is that, the particular patterns of social interactions between individuals matter. In other words, only looking at either individual behaviour or aggregate behaviour misses crucial aspects. Under this view, to understand the behaviour of individuals one has to understand the complex detail and dynamics of the interactions between them. Agent-based simulations, such as the one described does this as few other formal techniques do.

In particular, it allows for the integration of cognition and social processes, going beyond emergence and immergence to allow for a relatively 'tight' loop between the processes that go on in the head of an agent, the interactions between agents and their social structure. In agent-based simulations, these three things can be co-evolving, no aspect taking priority.

There has been a stream of models that aim to directly model the evolution of opinions within a group of interacting agents. These are the "opinion dynamics" (OD) models (e.g. [2]). In this the opinions of agents lie on a continuous line and these opinions directly effect each other when agents are sufficiently similar/near to each other on this scale. An example run of such a model is shown in Figure 6.

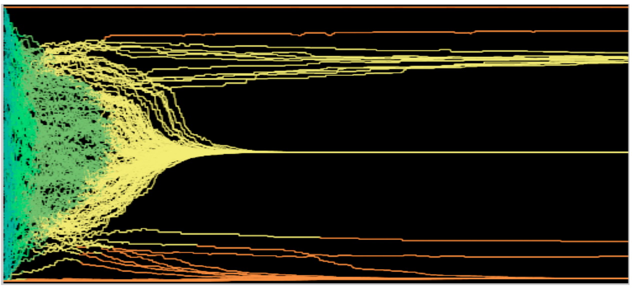

Figure 6. A typical evolution of opinions from an opinion dynamics model. Horizontal axis shows the strength of the opinions (from -1 = completely disagree, to 1 = completely agree). Time is along the x-axis. The colour shows the "certainty" of the agent from red = certain to blue = uncertain.

However, such models exhibit resultant behaviour that is very different from that we observe (for example during the Brexit vote). Firstly, the behaviour in OD models results in a number of stable groups that subsequently do not change. Secondly, typically all agents are influenced to increase certainty, so there are no completely uncertain agents left at the end. Thirdly, after a while, all opinions settle down to stable values. Finally, and more fundamentally, these models conflate the results of beliefs – measurable opinions – with the underlying mechanisms.

In this paper I wish to go behind such models. I want a model:

- Where agents are socially embedded with a tight loop between cognition and social structure
- That naturally develops a combination of some coherency within emergent groups but also with variety
- Where opinions do not settle down, but continue to change in a "noisy" fashion
- Where "opinions" do not directly act on each other but emerge from a meaningful discussion of belief

3. THE MODEL

The model to be described is quite abstract at the moment, *much* simpler than my usual style of model! It is merely a starting point, to point out that the intimate connection between the cognitive and the social can be represented and to stimulate discussion. I am looking for suitable data to enable its assessment and further development, and would welcome the suggstion of any rich data sets or qualitative research that I might compare this to.

In this model:

- There is a network of a fixed set of nodes and arcs (that can change)
- There are, *n*, different beliefs {A, B, ...} circulating between nodes
- Beliefs are copied along links or dropped by nodes according to the change in coherency of the node's belief set that this would result in
- Links can be randomly made
- Links are dropped when beliefs are rejected for copy between nodes

NODE PROPERTIES

Each node has:

- A (possibly empty) set of these "beliefs" that it holds
- A fixed "coherency" function from possible sets of beliefs to [-1, 1] where 1 is completely coherent, 0 is neutral and -1 is maximum incoherency.
- A fixed scaling function that maps changes in coherency to the probability of a change in beliefs
- A record of the last node it "rejected" a belief from

INITIALISATION

Beliefs and social structure are randomly initialized at the start according to some global parameters. In the present version there can be up to 3 types of agent, which are distinguished by their coherency and scaling function.

COHERENCY FUNCTION

Key to this model is the model of belief coherency, which is a generalisation of Thagard's pairwise (in)coherence. It gives a measure of the extent to which whole set of current beliefs are coherent. This assumes a background of shared beliefs which are not represented – this is important as the model only captures what might happen to a few foreground beliefs that are changing against all other beliefs. If one chose a different set of 'foreground' candidate beliefs from all possible beliefs then different coherency functions would be needed.

This allows for great flexibility in choices of belief structure, for example we could have the coherency evaluations: {A}➔0.5 and {B}➔{0.7} but also {A, B}➔-0.4 if beliefs A and B are mutually inconsistent, but individually coherent (against the background beliefs). The process of belief change is as follows.

> *The probability of gaining a new belief from another or dropping an existing belief in this model is monotonically dependent on whether it increases or decreases the coherency of the node's belief set*

BELIEF CHANGE PROCESSES

Each iteration the following occurs:

- *Copying*: each arc is selected; a source end and destination end selected; a belief at the source is randomly selected; then copied to the destination with a *probability* related to the change in coherency it would cause (due to the scaling function described next).
- *Dropping*: each node is selected; a random belief is selected and then dropped with a *probability* related to the change in coherency it would cause

SCALING IMPACT OF COHERENCY FUNCTION

There is a variety of ways to map a change in coherence to a probability (of a change occuring). The function that maps from changes in coherency to probability could be any that: (a) is monotonic (b) such that a -1➔1 change has probability of 1 (b) a

1➔-1 change has probability of 0. Two example such functions are illustrated in Figure 7.

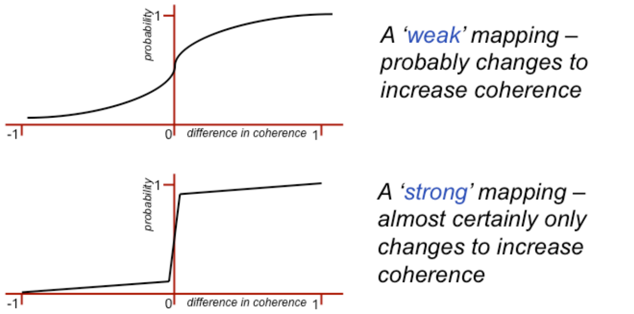

Figure 7. Two example mappings from a change in coherency to a probability (of either a "copy" or a "drop" of a belief).

The scaling function thus affects how amenable an agent is to change and the extent to which it may change. E.g. whethere only to increase coherency or if it can occasionally decrease.

NETWORK CHANGE PROCESSES

There are two processes for changing the influence network. Each iteration the following occurs for each agent:

- *Link Drop*: with a probability: if a belief copy was *rejected* by the recipient, then drop that in-link.
- *New Links*: with another probability, create a new random link with a random other (with a friend of a friend if possible, otherwise any)

OTHER

In order to maintain the average link density I added the following 'kludge': If there are too many links (as set by arcs-per-node) slightly increase the rate of link drop, if there are not enough, slightly reduce the rate of link drop. Also, nodes have to have at least one link, or one is added, to stop isolates forming.

Finally there is a small probability that a belief is randomly added or dropped, this adds a little bit of extrinsic noise into the system and stops beliefs disappearing (through chance) from the entire population (as discussed in [3]).

The "opinion" of agents is derived from the belief state of the agents. This is a function from the belief set to [-1, 1]. The global opinion is an average of this function applied to each agent. There is obviously a choice as to how this is done, but we do this uniformly.

More about the model can be found in the Appendix and [4].

4. PRELIMINARY RESULTS

If not otherwise mentioned parameter values for the runs below are those listed in the appendix. For purposes of exposition, beliefs are arbitrarily assigned colours and types of agents shapes. In both examples discussed:

- There are 50 agents
- Simulations last 1000 ticks
- There are, on average 3 arcs per node
- The copy-rate parameter is 0.3 (the probability that one belief is considered for being copied along any link)
- The drop-rate parameter is 0.075 (the probability that an agent considers dropping one belief in a tick)
- Probability of dropping a link is 0.2 (if there is a suitable candidate where a copy has been rejected)
- Probability of a random belief change is 0.001

The derived opinions reduce to the number of "blue" beliefs minus the number of "yellow" beliefs divided by the population.

AN ILLUSTRATIVE EXAMPLE WITH 3 BELIEFS AND 2 KINDS OF AGENT

This example helps give a flavour of the model. In this:

- There are 3 beliefs: "yellow", "blue" and "red"
- The probability of a new link is 0.01
- Two kinds of agent, a strong-minded minority (stars) and a more open-minded majority (circles).

The two kinds of agent are as follows:

- 20% of agents (stars) are such that the 'yellow' beliefs are attractive and the 'blue' ones unattractive (due to coherence with background beliefs), they are also 'strong minded' in the sense that they only change their mind if it increases their coherence (a strong mapping function)
- 80% of agents (circles) are such that the 'blue' beliefs are attractive and the 'yellow' ones unattractive, they are also 'weak minded' in the sense that they only have a tendency to change their mind if it increases their coherence (more probabilistic in their belief change)

Both kinds change their links (or not) similarly and both are agnostic with respect to the 'red' belief. Atomic beliefs: yellow, red, blue. Runs are initialised with random beliefs and network.

Here we try 10 runs of each variant: with no belief change and no link change; with belief change only; with link change only; and with both belief and link change. Output shown in terms of some typical runs and some summary graphs. In snapshots of the runs, agents shown in colours indicating the mixture of beliefs held (or if none, grey) – so blue if they only hold the Blue belief, green if they hold yellow and blue, etc.

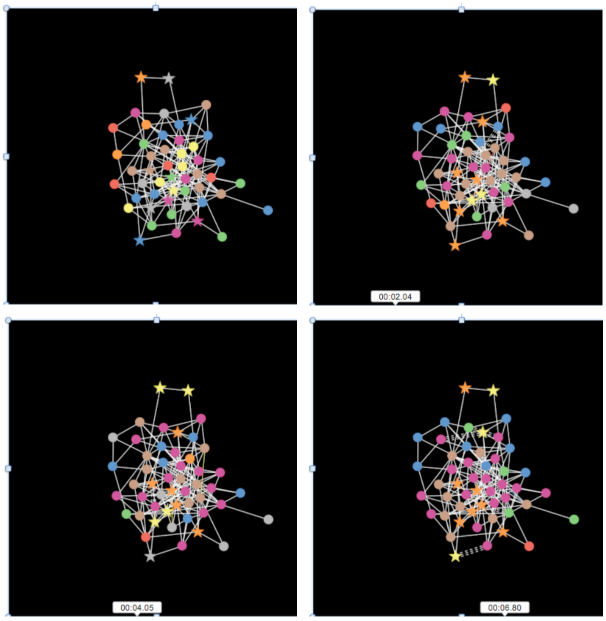

Figure 8. 4 snapshots of the run only allowing beliefs to change

When only beliefs are allowed to change, there is some sorting of agents, so agents who are connected are more likely to hold similar beliefs, but the fixed network structure limits the extent to which this can occur. There has been a shift in the balance of beliefs – a shift towards blue. (Figure 8)

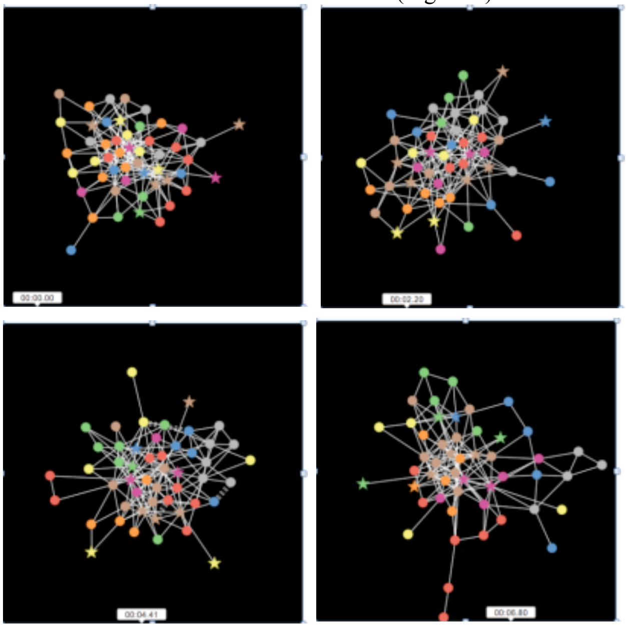

Figure 9. 4 snapshots of the run only allowing links to change

When only links are allowed to change, then some structure can evolve that partially separates agents with different beliefs. The balance between blue and yellow beliefs is largely unchanged (remember there is some random change). (Figure 9)

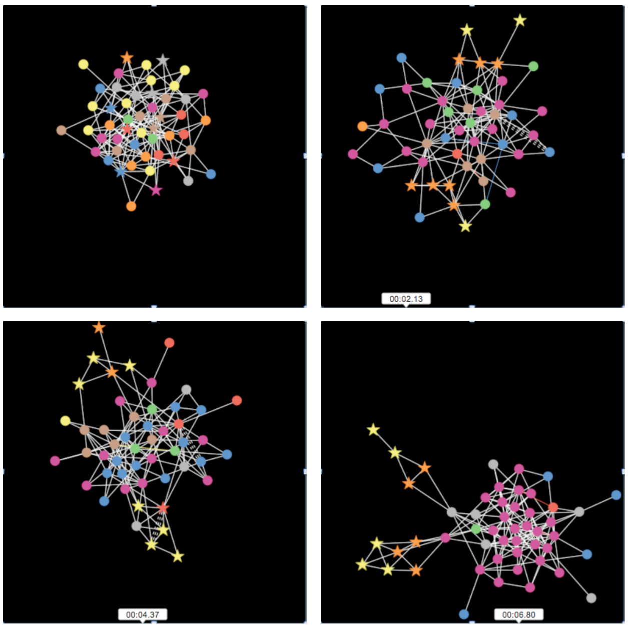

Figure 10. 4 snapshots of a run where both beliefs and links change

When both beliefs and network can change we find that a considerable sorting of kinds of agents has occurred, with the "stars" being marginalised. There has been a marked shift in overall opinion towards the blue (remember the purple are agents that hold the blue belief but also the red etc.). (Figure 10)

Figure 11 and Figure 12 show the average shift in belief averaged over 10 runs to 1000 ticks of the four model variants.

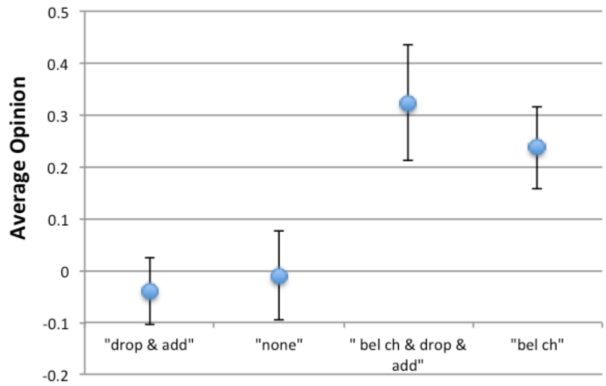

Figure 11. Average Opinion at end of four kinds of run, (where "drop & add" is link change, "bel ch" is belief change)

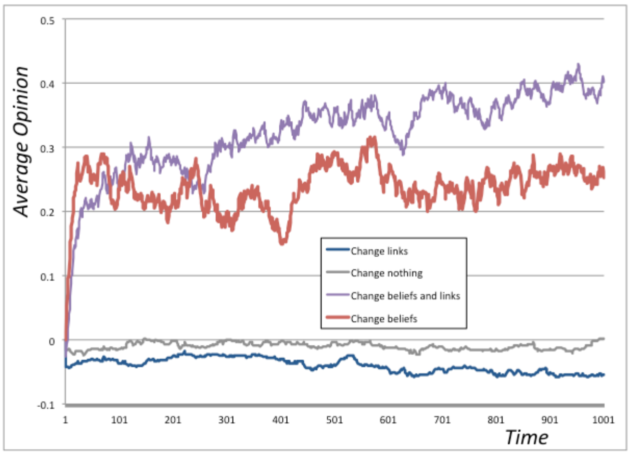

Figure 12. Opinion change, with and without changing links and beliefs (average over 10 runs)

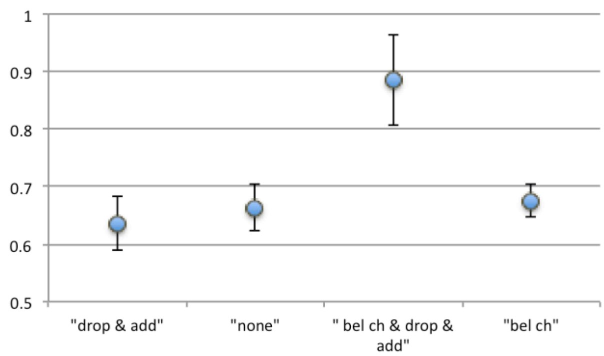

Figure 13. Proportion of same kinds together at end (av. over 10 runs, "drop & add" = link change, "bel ch" = belief change)

Figure 13 the proportion of links with the same kind of agent at its ends by the end of the simulations variants, averaged over 10 runs. This shows that the combination of belief change (a cognitive process) and link change (a social process) produces *qualitatively different results* from either belief change only or link change only.

TOWARDS A BREXIT EXAMPLE

In the second set of results we move towards the kind of situation that occurred in the Brexit referendum. Here we have 3 groups: floaters (most voters), yellows (Leave campaigners) and blues (Remain campaigners). Thus the model is composed of:

- 70% Floaters (circles), these are weakly positive towards having either yellow or blue beliefs, but not both. They have a weak scaling function so they are more open to change and more tolerant of temporarily tolerating moves to lower coherence
- 10% Leavers (stars) are for yellow and against blue with a strong scaling function (leave)
- 20% Remainers (triangles) are for blue and against yellow, with a medium scaling function (remain)

Groups start separate (to allow for self-reinforcement), so that initially nodes are only linked with others of their type. They are initialised with random beliefs.

There are two beliefs: blue and yellow, the opinion function is the same as before (blue-yellow). For some reason that I cannot recall, the probability of a new link is 0.025 in these runs.

There are no variants of the run (other than having different random seeds), all runs have the same proportions of agents, kind of initialization and parameters.

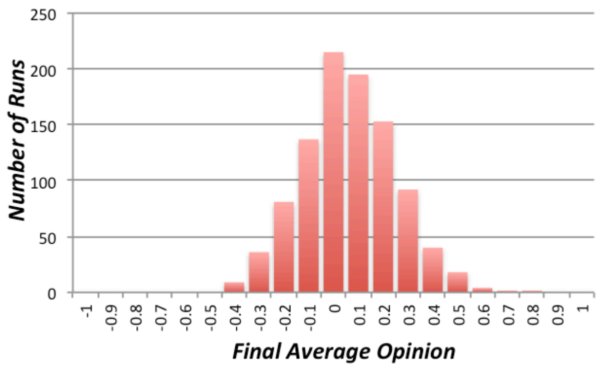

Figure 14. Distribution of final average opinions over 1000 runs.

When the simulation is run 1000 times, we get the distribution of final average opinonis as shown in Figure 14. One can see that, given this set-up there is a slight overall bias towards blue (remain) outcomes.

Below I will merely display 4 specific runs to show the kinds of dynamics that this model can display. For each of these I will show 4 snapshots of the state of the model (as before) plus a graph that shows how the average opinion changed over time.

Example run 1

In this run the stars connect with floaters first, followed by the circles which has the effect of polarising the floaters, which then separate off into two groups. In each of these groups, the campaigners slowly convert the floaters to their own colour. One can see how the campaigners of both sides are now embedded within tightly formed groups (Figure 15). The average opinion oscillates between blue and yellow over time but the blues gather the biggest group in the end and win (Figure 16).

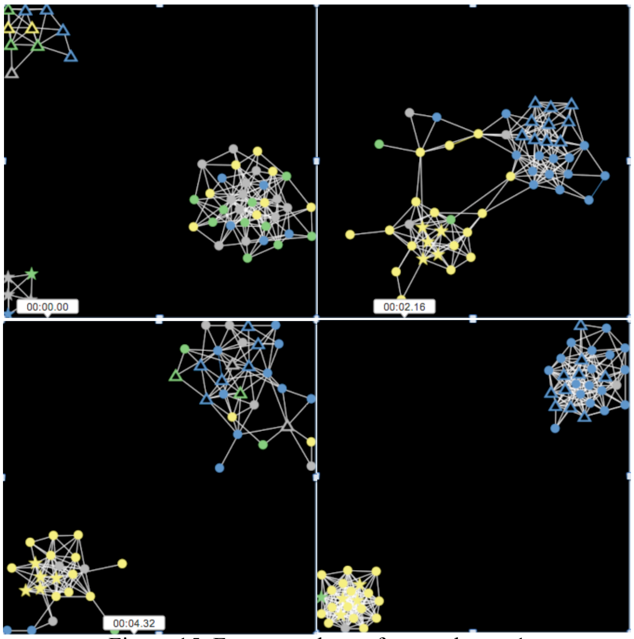

Figure 15. Four snapshots of example run 1

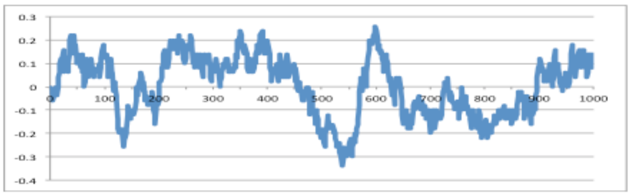

Figure 16. The changing average opinion in example run 1

Example run 2

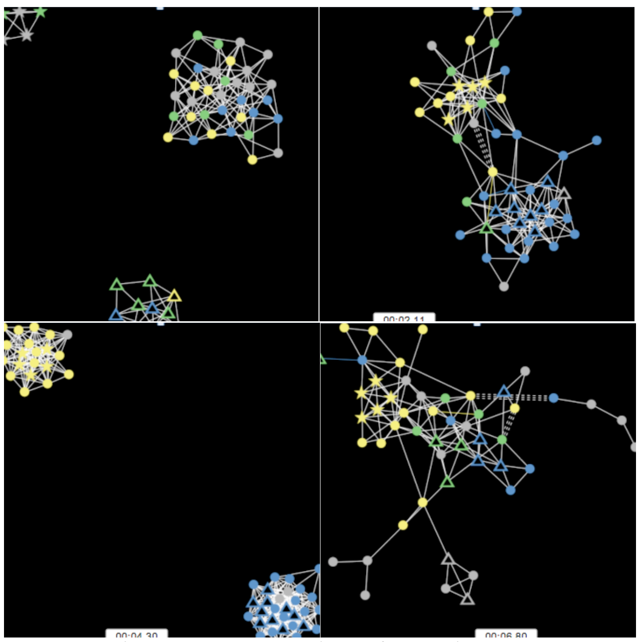

Figure 17. Four snapshots of example run 2

In the second run a similar thing happens, two groups are established (earlier) which are stable for a while. Then some yellow mutate within the blue island, which then pulls apart. The yellow group then integrates into other and converts some more of the floaters (Figure 17). Figure 18 shows the stability of the overall opinion for blue until towards the end where the yellow group manage to connect into the main group and shift some more floaters to their side, winning in the end.

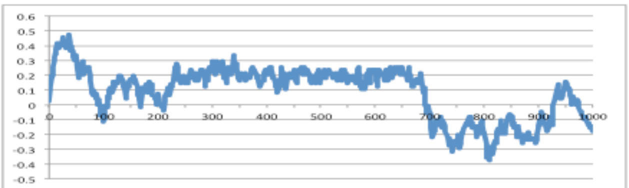

Figure 18. The changing average opinion in example run 2

Example run 3

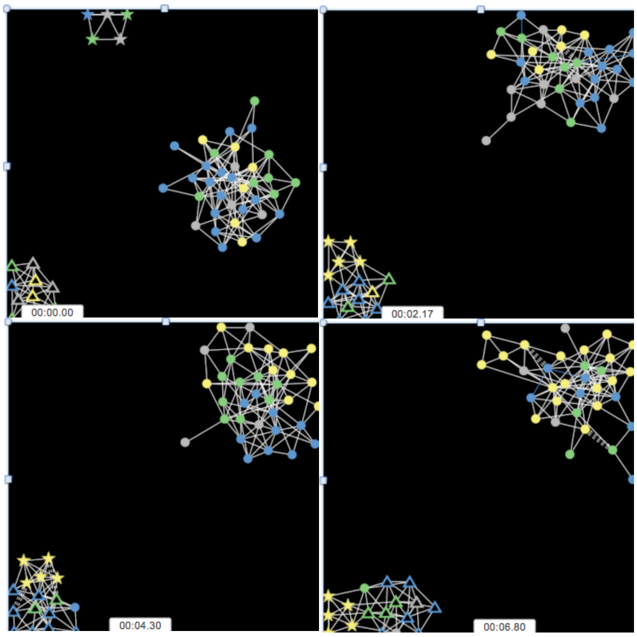

Figure 19. Four snapshots of example run 3

In this run, we have a ‘Westminster bubble’ of stars and triangles forming, separate from the floaters (Figure 19). Free from influence from campaigners, floaters flip each other back and forth but in a random walk, which happens to end with more yellow (Figure 20).

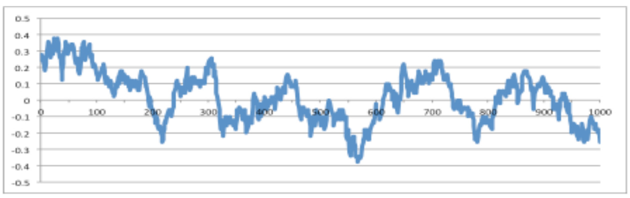

Figure 20. The changing average opinion in example run 3

Example run 4

In this run, both starts and triangles connect with floaters, but triangles more intimately, stars always on the peripheral. The yellow stars connect repeatedly to the main group but are isolated again each time, so they do not ever gain enough influence to convert many floaters (Figure 21). The average opinion oscillates but always on the blue side (Figure 22).

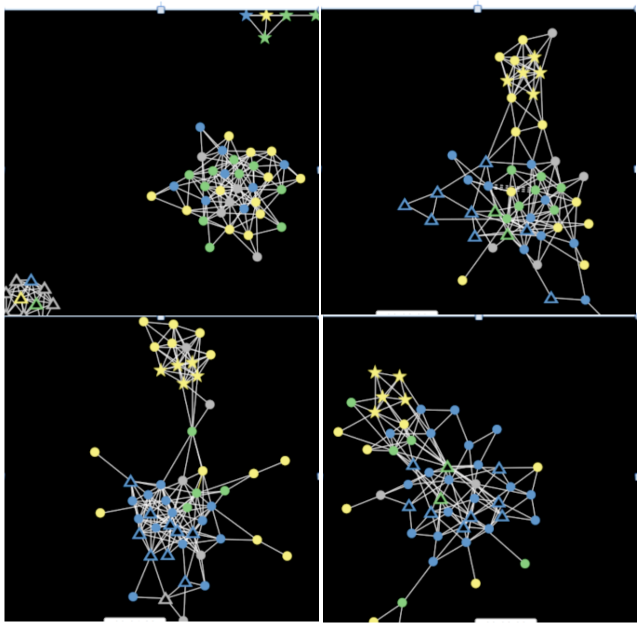

Figure 21. Four snapshots of example run 4

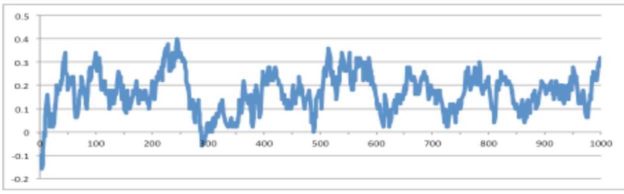

Figure 22. The changing average opinion in example run 4

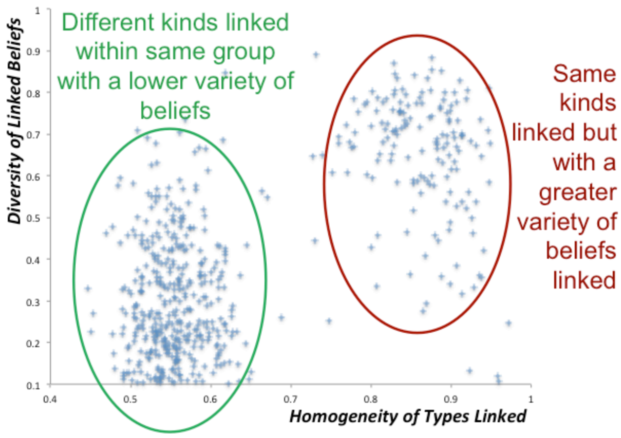

Figure 23. A scatter graph of the diversity of beliefs between linked nodes and the homogeneity of types of agents linked at the end of 1000 runs of 1000 ticks.

If one measures the end diversity of beliefs in linked nodes and the homogeneity of types (circles etc.) linked over 1000 runs one ends up with two distinct clusters of outcome (Figure 23).

5. DISCUSSION

In this model, here are 'competing' processes of social influence (suggestion) vs. internal coherence with existing set of beliefs; also between social influence vs. social linking. Thus, an 'extreme' group may be good at convincing another group when well connected to that group but groups tend to disconnect from those with very different views to themselves.

How processes actually happen may matter a lot, so it may be that this model has these wrong. We just do not know what influences people's change of links – do people have a 'whitelist' of those they are willing to allow to influence them? In addition, this model does not touch upon the development of people's belief structures within their society during acculturation or youth.

However, this model does suggest possibilities.It may be that how we act collectively is not through a direct spread (imitation) of action but via a spread of beliefs, norms, stories, habits etc. from which directed action springs. This would allow for specialisation and diversity of action whilst maintaining coherency. In this picture culture (any pattern, knowledge, norms, technology passed down the generations etc.) is important, but the group-structure of society is dynamic and can be complex (almost fractal in structure).

It does vividly show that if one modelled only the belief change/influence processes or only the social network processes then one could be missing significant aspects of what might be happening during these complex socio-cognitive processes.

Figure 24. A tethered goat

Like a tethered goat (Figure 24), individuals may find it hard to wander too far from the beliefs of the group it is attached to, but we might be able to choose to which group we are tied.

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APPENDIX – MORE ABOUT THE MODEL

This model is an extension of the one discussed in [3], but network change processes. The full model has quite a number of options and extensions not discussed here. The full model can be downloaded and inspected from OpenABM [4].

PARAMETERS

A full list of parameters with descriptions maybe be found in the "Info" tab of the model [4]. Important parameters for our purposes include the following (the default value is shown in brackets following the description).

- *num-agents* – number of agents in the simulation (50)
- *num-beliefs* – number of atomic beliefs around (2)
- *init-prob-belief* – probability that agents hold each of the atomic beliefs at the start (0.5)
- *copy-prob* – the probability that a (random) belief from one agent will be attempted to be copied to anther during the copy process (0.3)
- *drop-rate* – the drop-rate is the probability that an individual will do the drop process once (per simulation tick) (0.075)
- *mut-prob-power* – this is the power of 10 of the probability that a random belief of an agent is flipped each time click (so -3 is a probability of 0.001)
- *arcs-per-node* – how many arcs lead into each node on average (3)
- *init-sep-prob* – the probability that types only link to their own kind at the start (1)
- *init-prob-drop-link* – the probability of dropping a link (0.2)
- *prob-new-link* –this is probability of adding a new random link (0.025)
- *Opinion-Fn-Name* – The function that is used for recovering the opinion from the beliefs of agents which is then averaged for the global Opinion (blue-yellow)